Implications of Pseudospin Symmetry on Relativistic Magnetic Properties and Gamow - Teller Transitions in Nuclei

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Abstract

Recently it has been shown that pseudospin symmetry has its origins in a relativistic symmetry of the Dirac Hamiltonian. Using this symmetry we relate single - nucleon relativistic magnetic moments of states in a pseudospin doublet to the relativistic magnetic dipole transitions between the states in the doublet, and we relate single - nucleon relativistic Gamow - Teller transitions within states in the doublet. We apply these relationships to the Gamow - Teller transitions from ^{39}Ca to its mirror nucleus ^{39}K.

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I. INTRODUCTION

For nucleons moving in a relativistic mean field with scalar V_S and vector potentials V_V, an SU(2) symmetry exists for the case for which $V_S = -V_V$ [1]. This symmetry manifests itself in nuclei as a slightly broken symmetry [2–5] since $|\frac{V_S+V_V}{V_S-V_V}|$ is small for realistic mean fields [6–10], and, in fact, gives rise to what has been called "pseudospin symmetry". The original observations that led to the coining of the word "pseudospin symmetry" were quasi-degeneracies in spherical shell model orbitals with non - relativistic quantum numbers (n_r, ℓ, $j = \ell+1/2$) and ($n_r-1, \ell+2$, $j = \ell+3/2$) where n_r, ℓ, and j are the single-nucleon radial, orbital, and total angular momentum quantum numbers, respectively [11,12]. This doublet structure is expressed in terms of a "pseudo" orbital angular momentum $\tilde{\ell} = \ell + 1$, the average of the orbital angular momentum of the two states in doublet, and "pseudo" spin, $\tilde{s} = 1/2$. For example, $(n_r s_{1/2}, (n_r-1)d_{3/2})$ will have $\tilde{\ell} = 1$, $(n_r p_{3/2}, (n_r-1)f_{5/2})$ will have $\tilde{\ell} = 2$, etc. These doublets are almost degenerate with respect to pseudospin, since $j = \tilde{\ell} \pm \tilde{s}$ for the two states in the doublet; examples are shown in Figure 1. Pseudospin "symmetry" was shown to exist in deformed nuclei as well [13,14] and has been used to explain features of deformed nuclei, including superdeformation [15] and identical bands [16,17]. However, the origin of pseudospin symmetry remained a mystery and "no deeper understanding of the origin of these (approximate) degeneracies" existed [18]. A few years ago it was shown that relativistic mean field theories gave approximately the correct spin orbit splitting to produce the pseudospin doublets [19]. Finally the source of pseudospin symmetry as a broken symmetry of the Dirac Hamiltonian related to $V_S \approx -V_V$ was pointed out [2–5]. For spherical nuclei, pseudo-orbital angular momentum $\tilde{\ell}$ is also conserved and physically is the "orbital angular momentum" of the lower component of the Dirac wavefunction.

One consequence of this relativistic SU(2) pseudospin symmetry is that the spatial wave-function for the lower component of the Dirac wavefunctions will be equal in shape and

magnitude for the two states in the doublet [3–5]. For spherical nuclei, this means that the radial wavefunctions for the lower components in the doublet will have the same number of nodes, so we label these states with pseudo-radial quantum number (i.e.; the radial quantum number of the lower component ($\tilde{n} = 0, 1, \ldots$)). Furthermore,the pseudo-orbital angular momentum will be a conserved quantum number for spherical symmetric scalar and vector potentials and so we label the states with the pseudo-orbital angular momentum $\tilde{\ell}$ [4]. Finally, the total angular momentum j ($\vec{j} = \vec{\tilde{\ell}} + \vec{\tilde{1}}/2$), and projection m, are conserved as well. The Dirac wavefunction for the two states in the doublet are

$$\Psi_{\tilde{n},\tilde{\ell},j=\tilde{\ell}+1/2,m} = (g_{\tilde{n}-1,\tilde{\ell},j}[Y_{\tilde{\ell}+1}\chi]_m^{j=\tilde{\ell}+1/2}, i f_{\tilde{n},\tilde{\ell},j}[Y_{\tilde{\ell}}\chi]_m^{j=\tilde{\ell}+1/2}),$$

$$\Psi_{\tilde{n},\tilde{\ell},j=\tilde{\ell}-1/2,m} = (g_{\tilde{n},\tilde{\ell},j}[Y_{\tilde{\ell}-1}\chi]_m^{(j=\tilde{\ell}-1/2)}, i f_{\tilde{n},\tilde{\ell},j}[Y_{\tilde{\ell}}\chi]_m^{(j=\tilde{\ell}-1/2)}), \tag{1}$$

where g, f are the radial wave functions, $Y_{\tilde{\ell}}$ are the spherical harmonics, χ is a two-component Pauli spinor, and $[\ldots]^{(j)}$ means coupled to angular momentum j. We note that the upper component of the $j = \tilde{\ell} - 1/2$ wavefunction has the same radial quantum number as the lower component, whereas the upper component of the $j = \tilde{\ell} + 1/2$ wavefunction has radial quantum number one unit less than the lower component. The normalization of the wavefunction gives

$$\int_0^\infty [g^2_{\tilde{n}',\tilde{\ell},j} + f^2_{\tilde{n},\tilde{\ell},j}] r^2 dr = 1;$$

$$j = \tilde{\ell} + 1/2,\ \tilde{n}' = \tilde{n} - 1;\ j = \tilde{\ell} - 1/2,\ \tilde{n}' = \tilde{n}. \tag{2}$$

For a square well potential, the overall phase between the two amplitudes will be a minus sign [2] so we expect that, in the symmetry limit for realistic potentials, $f_{\tilde{n},\tilde{\ell},j=\tilde{\ell}+1/2}(r) = -f_{\tilde{n},\tilde{\ell},j=\tilde{\ell}-1/2}(r) = f_{\tilde{n},\tilde{\ell}}(r)$. For the relativistic mean field approximation to relativistic Lagrangrians with realistic zero range interactions and to nuclear field theory with meson exchanges it was indeed shown that, $f_{\tilde{n},\tilde{\ell},j=\tilde{\ell}+1/2}(r) \approx -f_{\tilde{n},\tilde{\ell},j=\tilde{\ell}-1/2}(r)$ [3,10].

However, to date, the effect of pseudospin symmetry on the relativistic wavefunction has not been tested empirically. Since the lower component of the Dirac wavefunction is small [3,5,10] this effect will be difficult to detect except perhaps in certain forbidden transitions. For example, single - nucleon magnetic dipole and Gamow-Teller transitions between pseudospin doublets are forbidden non-relativistically (i.e., "ℓ forbidden" [20,21]) because the orbital angular momenta of the two states differ by two units. However, they are not forbidden relativistically. In this paper we shall use approximate pseudospin symmetry in the wavefunction to derive relations between single-nucleon relativistic magnetic moments and magnetic dipole transtions within a pseudospin doublet on the one hand, and between single-nucleon relativistic Gamow-Teller transitions within a pseudospin doublet on the other hand. These relationships provide a test for the influence of pseudospin symmetry on the single - nucleon wavefunctions.

II. MAGNETIC MOMENTS AND TRANSITIONS

The relativistic magnetic dipole operator for a particle with charge e is given by [22,23],

$$\hat{\mu}_i = -\frac{e}{2}\, g_\rho\, (\vec{\alpha} \times \vec{r})_i + \mu_{A,\rho}\, \sigma_i, \tag{3}$$

where $\vec{\alpha}$ is the usual Dirac matrix, $\vec{r}$ is the three space vector, $\rho = \pi$ for a proton and ν for a neutron, g_ρ is the orbital gyromagnetic ratio, $g_\pi = 1, g_\nu = 0$, and $\mu_{A,\rho}$ is the anamolous magnetic moment, $\mu_{A,\pi} = 1.793\mu_0$, $\mu_{A,\nu} = -1.913\mu_0$, where $\mu_0 = \frac{e\hbar}{2Mc}$ is the nuclear magneton. The magnetic moment is given in terms of the matrix element of this operator with m = j,

$$\mu_{j,\rho} = \langle \Psi_{\tilde{n},\tilde{\ell},j,m=j,\rho} | \hat{\mu} | \Psi_{\tilde{n},\tilde{\ell},j,m=j,\rho} \rangle, \tag{4}$$

and the square root of the magnetic transition probability between two states in the doublet is given in terms of the reduced matrix element of this operator,

$$\sqrt{B(M1:\tilde{n},\tilde{\ell},j'\rightarrow\tilde{n},\tilde{\ell},j)_\rho}=\frac{1}{(2j'+1)}\langle\Psi_{\tilde{n}',\tilde{\ell},j',\rho}||\hat{\mu}||\Psi_{\tilde{n},\tilde{\ell},j,\rho}\rangle \tag{5}$$

Using the Dirac wavefunction (1), this results in

$$j=\tilde{\ell}-1/2$$

$$\mu_{j,\rho}=\frac{-e\ g_\rho\ (j+1/2)}{2(j+1)}\int_0^\infty g_{\tilde{n},\tilde{\ell},j}f_{\tilde{n},\tilde{\ell},j,\rho}\ r^3\ dr+\mu_{A,\rho}\ (1-\frac{(2j+1)}{(j+1)}\ \int_0^\infty f^2_{\tilde{n},\tilde{\ell},j,\rho}\ r^2\ dr)\ , \tag{6}$$

$$j=\tilde{\ell}+1/2$$

$$\mu_{j,\rho}=\frac{e\ g_\rho(j+1/2)}{2(j+1)}\int_0^\infty g_{\tilde{n}-1,\tilde{\ell},j,\rho}f_{\tilde{n},\tilde{\ell},j,\rho}\ r^3\ dr-\frac{\mu_{A,\rho}}{(j+1)}\ (j-(2j+1)\ \int_0^\infty f^2_{\tilde{n},\tilde{\ell},j,\rho}\ r^2\ dr)\ , \tag{7}$$

$$j'=\tilde{\ell}+1/2,\ \ j=\tilde{\ell}-1/2$$

$$\sqrt{B(M1:\tilde{n},\tilde{\ell},j'\rightarrow\tilde{n},\tilde{\ell},j)_\rho}=-\sqrt{\frac{(2j+1)}{(2j+3)}}\sqrt{B(M1:\tilde{n},\tilde{\ell},j\rightarrow\tilde{n},\tilde{\ell},j')_\rho}=$$

$$-\frac{1}{4}\sqrt{\frac{(2j+1)}{(j+1)}}\ [\frac{e\ g_\rho}{2}\ \int_0^\infty[g_{\tilde{n}-1,\tilde{\ell},j',\rho}f_{\tilde{n},\tilde{\ell},j,\rho}+g_{\tilde{n},\tilde{\ell},j,\rho}f_{\tilde{n},\tilde{\ell},j',\rho}]\ r^3\ dr+4\mu_{A,\rho}\ \int_0^\infty f_{\tilde{n},\tilde{\ell},j',\rho}f_{\tilde{n},\tilde{\ell},j,\rho}\ r^2\ dr\ ]. \tag{8}$$

A. Non-relativistic Limit

The Dirac equation with speherically symmetric potentials reduces to two coupled one - dimensional radial equations for the upper and lower components, (g,f) [2],

$$\hbar\ c[\frac{d}{dr}+\frac{1+\kappa}{r}]g_{\tilde{n}',\tilde{\ell},j,\rho}=[2Mc^2-E+V_S-V_V]\ f_{\tilde{n},\tilde{\ell},j,\rho}, \tag{9}$$

$$\hbar\ c[\frac{d}{dr}+\frac{1-\kappa}{r}]f_{\tilde{n},\tilde{\ell},j,\rho}=[E+V_S+V_V]\ g_{\tilde{n}',\tilde{\ell},j,\rho}, \tag{10}$$

where

$$\kappa=-\tilde{\ell},j=\tilde{\ell}-1/2;\quad \kappa=\tilde{\ell}+1,j=\tilde{\ell}+1/2, \tag{11}$$

M is the nucleon mass, and E is the binding energy. In order to determine $\int_0^\infty gfr^3\ dr$ we use (9, 10) to derive [22]:

$$g_{\tilde{n}',\tilde{\ell},j',\rho} f_{\tilde{n},\tilde{\ell},j,\rho} =$$

$$\frac{\hbar c}{2Mc^2 + 2V_S} \left[g_{\tilde{n}',\tilde{\ell},j',\rho} \frac{d}{dr} g_{\tilde{n}',\tilde{\ell},j,\rho} + f_{\tilde{n},\tilde{\ell},j',\rho} \frac{d}{dr} f_{\tilde{n},\tilde{\ell},j,\rho} + \frac{1+\kappa}{r} g_{\tilde{n}',\tilde{\ell},j',\rho} g_{\tilde{n}',\tilde{\ell},j,\rho} + \frac{1-\kappa}{r} f_{\tilde{n},\tilde{\ell},j',\rho} f_{\tilde{n},\tilde{\ell},j,\rho} \right] \tag{12}$$

In the non-relativistic limit, the potentials are ignored with respect to the nucleon mass, although $\frac{V_S}{Mc^2} \approx .48$ in the interior of the nucleus. Also terms quadratic in f are ignored. This gives

$$\int_0^\infty r^3 \; dr [g_{\tilde{n}',\tilde{\ell},j',\rho} f_{\tilde{n},\tilde{\ell},j,\rho} + g_{\tilde{n}',\tilde{\ell},j,\rho} f_{\tilde{n},\tilde{\ell},j',\rho}] = \frac{\hbar}{2Mc} (\kappa + \kappa' - 1) \int_0^\infty r^2 \; dr g_{\tilde{n}',\tilde{\ell},j',\rho} \; g_{\tilde{n}',\tilde{\ell},j,\rho}. \tag{13}$$

For $j' = j$, $\int_0^\infty r^3 \; dr g_{\tilde{n}',\tilde{\ell},j,\rho} \; g_{\tilde{n}',\tilde{\ell},j,\rho}$ =1 from the normalization condition (2). Therefore in the non-relativistic limit, the magnetic moments become,

$$\mu_{j,\rho} = (j + 1/2) \; g_\rho \; \mu_0 + \mu_{A,\rho}; \; j = \tilde{\ell} - 1/2, \tag{14}$$

$$\mu_{j,\rho} = \frac{j}{(j+1)} ((j + 1/2) \; g_\rho \; \mu_0 - \mu_{A,\rho}); \; j = \tilde{\ell} + 1/2. \tag{15}$$

The non-relativistic limits for the magnetic moments in (14,15) are equivalent to the Schmidt values [24].

However, for $j' \neq j$, it follows from (11) that $\kappa + \kappa' - 1 = 0$ and therefore,

$$B(M1 : \tilde{n}, \tilde{\ell}, j' \rightarrow \tilde{n}, \tilde{\ell}, j)_\rho = 0; \; j' \neq j, \tag{16}$$

Thus the non-relativistic limit of the B(M1) is zero which is as it should be since the transition is from ℓ to $\ell \pm 2$ as stated in the Introduction.

B. Pseudospin Symmetry

Instead of looking at the non-relativistic limit, we examine the pseudospin limit which assumes that the spatial wave functions of the lower components of the doublet are equal and opposite in sign,

$$f_{\tilde{n},\tilde{\ell},j=\tilde{\ell}+1/2,\rho}(r) = -f_{\tilde{n},\tilde{\ell},j=\tilde{\ell}-1/2,\rho}(r) = f_{\tilde{n},\tilde{\ell},\rho}(r). \tag{17}$$

Inserting this relation into (6, 7, 8) we obtain,

$$j = \tilde{\ell} - 1/2$$

$$\mu_{j,\rho} = \frac{e\ g_\rho\ (j+1/2)}{2(j+1)} \int_0^\infty g_{\tilde{n},\tilde{\ell},j,\rho} f_{\tilde{n},\tilde{\ell},\rho}\ r^3\ dr + \mu_{A,\rho}\ (1 - \frac{(2j+1)}{(j+1)}\ \int_0^\infty f^2_{\tilde{n},\tilde{\ell},\rho}\ r^2\ dr)\ , \tag{18}$$

$$j = \tilde{\ell} + 1/2$$

$$\mu_{j,\rho} = \frac{e\ g_\rho (j+1/2)}{2(j+1)} \int_0^\infty g_{\tilde{n}-1,\tilde{\ell},j,\rho} f_{\tilde{n},\tilde{\ell},\rho}\ r^3\ dr - \frac{\mu_{A,\rho}}{(j+1)}\ (j - (2j+1)\ \int_0^\infty f^2_{\tilde{n},\tilde{\ell},\rho}\ r^2\ dr)\ , \tag{19}$$

$$j' = \tilde{\ell} + 1/2,\ j = \tilde{\ell} - 1/2$$

$$\sqrt{B(M1:\tilde{n},\tilde{\ell},j' \to \tilde{n},\tilde{\ell},j)_\rho} = -\sqrt{\frac{(2j+1)}{(2j+3)}}\sqrt{B(M1:\tilde{n},\tilde{\ell},j \to \tilde{n},\tilde{\ell},j')_\rho} =$$

$$-\frac{1}{4}\sqrt{\frac{(2j+1)}{(j+1)}}\ [\frac{e\ g_\rho}{2}\ \int_0^\infty [-g_{\tilde{n}-1,\tilde{\ell},j',\rho} + g_{\tilde{n},\tilde{\ell},j,\rho}] f_{\tilde{n},\tilde{\ell},\rho}]\ r^3\ dr - 4\mu_{A,\rho}\ \int_0^\infty f^2_{\tilde{n},\tilde{\ell},\rho}\ r^2\ dr\ ]. \tag{20}$$

For neutrons $g_\nu = 0$, and hence we have one unkown quantity, $\int_0^\infty f^2_{\tilde{n},\tilde{\ell},\rho}\ r^2\ dr$. Therefore, if we know one magnetic quantity, we can predict two others,

$$\sqrt{B(M1:\tilde{n},\tilde{\ell},j' \to \tilde{n},\tilde{\ell},j)_\nu} = -\sqrt{\frac{j+1}{2j+1}}(\mu_{j,\nu} - \mu_{A,\nu}), \tag{21}$$

$$\sqrt{B(M1:\tilde{n},\tilde{\ell},j' \to \tilde{n},\tilde{\ell},j)_\nu} = \frac{j+2}{2j+3}\ \sqrt{\frac{2j+1}{j+1}}(\mu_{j',\nu} + \frac{j+1}{j+2}\mu_{A,\nu}). \tag{22}$$

For protons there are three unkown integrals, and so we can only derive one relationship between the three magnetic quantities,

$$\sqrt{B(M1:\tilde{n},\tilde{\ell},j' \to \tilde{n},\tilde{\ell},j)_\pi} = \frac{((j+2)(2j+1)\mu_{j',\pi} - (2j+3)(j+1)\mu_{j,\pi} + 4\ (j+1)^2\ \mu_{A,\pi})}{2\ (2j+3)\sqrt{(j+1)(2j+1)}};$$

$$j' = \tilde{\ell} + 1/2,\ j = \tilde{\ell} - 1/2. \tag{23}$$

If the magnetic moments are given by the Schmidt values as in (14, 15), then the magnetic transitions in (21, 22, 23) will be identically zero, which is consistent with the non-relativistic limit.

The relativistic mean field overestimates the isoscalar magnetic moments of nuclei [23]. However, when the response of the spectator nucleons is included, the relativistic isoscalar magnetic moments agree better with experiment [25]. The response of the spectator nucleons do not significantly affect isovector magnetic moments since the dominant mesons in the relativistic field theory are isoscalar. If we define the isoscalar and vector operators as

$$\mu_{j,S} = \frac{1}{2}(\mu_{j,\nu} + \mu_{j,\pi}); \mu_{j,V} = \frac{1}{2}(\mu_{j,\nu} - \mu_{j,\pi}); \mu_{A,S} = \frac{1}{2}(\mu_{A,\nu} + \mu_{A,\pi}); \mu_{A,V} = \frac{1}{2}(\mu_{A,\nu} - \mu_{A,\pi});$$

$$\sqrt{B(M1:\tilde{n},\tilde{\ell},j' \to \tilde{n},\tilde{\ell},j)}_S = \frac{1}{2}(\sqrt{B(M1:\tilde{n},\tilde{\ell},j' \to \tilde{n},\tilde{\ell},j)}_\nu + \sqrt{B(M1:\tilde{n},\tilde{\ell},j' \to \tilde{n},\tilde{\ell},j)}_\pi);$$

$$\sqrt{B(M1:\tilde{n},\tilde{\ell},j' \to \tilde{n},\tilde{\ell},j)}_V = \frac{1}{2}(\sqrt{B(M1:\tilde{n},\tilde{\ell},j' \to \tilde{n},\tilde{\ell},j)}_\nu - \sqrt{B(M1:\tilde{n},\tilde{\ell},j' \to \tilde{n},\tilde{\ell},j)}_\pi), \tag{24}$$

then the relations are separated into relations among the isoscalar and isovector magnetic properties:

$$\sqrt{B(M1:\tilde{n},\tilde{\ell},j' \to \tilde{n},\tilde{\ell},j)}_{S/V} = \frac{((j+2)(2j+1)\mu_{j',S/V} - (2j+3)(j+1)\mu_{j,S/V} + 4\ (j+1)^2\ \mu_{A,S/V})}{2\ (2j+3)\sqrt{(j+1)(2j+1)}};$$

$$j' = \tilde{\ell} + 1/2,\ \ j = \tilde{\ell} - 1/2. \tag{25}$$

III. GAMOW - TELLER TANSITIONS

The Gamow - Teller operator is given by

$$GT = \frac{g_A}{\sqrt{2}}\ \sigma\tau_\pm, \tag{26}$$

where g_A is the axial vector coupling constant (= 1.2670 (35)) and $\tau_\pm$ are the isospin raising and lowering operator. Thus this operator is a pure isovector operator. Using the Dirac wavefunction (1), this results in

$$j = \tilde{\ell} - 1/2$$

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j,\rho\to\tilde{n},\tilde{\ell},j,\bar{\rho})}=\sqrt{\frac{(j+1)}{j}}\ g_A\ (1-\frac{(2j+1)}{(j+1)}\ \int_0^\infty f_{\tilde{n},\tilde{\ell},j,\rho}f_{\tilde{n},\tilde{\ell},j,\bar{\rho}}\ r^2\ dr)\ , \quad (27)$$

$$j=\tilde{\ell}+1/2$$

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j,\rho\to\tilde{n},\tilde{\ell},j,\bar{\rho})}=-\frac{g_A}{\sqrt{j(j+1)}}\ (j-(2j+1)\ \int_0^\infty f_{\tilde{n},\tilde{\ell},j,\rho}f_{\tilde{n},\tilde{\ell},j,\bar{\rho}}\ r^2\ dr)\ , \quad (28)$$

$$j'=\tilde{\ell}+1/2,\ \ j=\tilde{\ell}-1/2$$

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j',\rho\to\tilde{n},\tilde{\ell},j,\bar{\rho})}=-\sqrt{\frac{(2j+1)}{(2j+3)}}\sqrt{B(GT:\tilde{n},\tilde{\ell},j,\bar{\rho}\to\tilde{n},\tilde{\ell},j',\rho)}=$$

$$-\sqrt{\frac{(2j+1)}{j+1}}\ g_A\ \int_0^\infty f_{\tilde{n},\tilde{\ell},j',\rho}f_{\tilde{n},\tilde{\ell},j,\bar{\rho}}\ r^2\ dr\ . \quad (29)$$

where $\bar{\rho}=\pi$ if $\rho=\nu$ and $\bar{\rho}=\nu$ if $\rho=\pi$.

We notice that

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j,\rho\to\tilde{n},\tilde{\ell},j,\bar{\rho})}=\sqrt{B(GT:\tilde{n},\tilde{\ell},j,\bar{\rho}\to\tilde{n},\tilde{\ell},j,\rho)}, \quad (30)$$

but, in general,

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j',\rho\to\tilde{n},\tilde{\ell},j,\bar{\rho})}\neq\sqrt{B(GT:\tilde{n},\tilde{\ell},j',\bar{\rho}\to\tilde{n},\tilde{\ell},j,\rho)}, \quad (31)$$

A. Non-Relativistic Limit of the Gamow - Teller Transitions

Since terms quadratic in f are ignored in the non- relatvistic limit, we get the usual results,

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j,\rho\to\tilde{n},\tilde{\ell},j,\bar{\rho})}=\sqrt{\frac{(j+1)}{j}}\ g_A;\ \ j=\tilde{\ell}-1/2, \quad (32)$$

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j,\rho\to\tilde{n},\tilde{\ell},j,\bar{\rho})}=-\sqrt{\frac{j}{(j+1)}}\ g_A;\ \ j=\tilde{\ell}+1/2 \quad (33)$$

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j',\rho\to\tilde{n},\tilde{\ell},j,\bar{\rho})}=0;\ \ j'\neq j \quad (34)$$

B. Pseudospin Symmetry

Using pseudospin symmetry, (17), there is only one unkown for the Gamow - Teller transtions and hence each transition is related to the other,

$$j' = \tilde{\ell} + 1/2, \;\; j = \tilde{\ell} - 1/2.$$

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j',\rho\rightarrow\tilde{n},\tilde{\ell},j,\bar{\rho})} = -\sqrt{\frac{j}{2j+1}}(\sqrt{B(GT:\tilde{n},\tilde{\ell},j,\rho\rightarrow\tilde{n},\tilde{\ell},j,\bar{\rho})} - \sqrt{\frac{j+1}{j}}\; g_A), \tag{35}$$

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j',\rho\rightarrow\tilde{n},\tilde{\ell},j,\bar{\rho})} = \frac{\sqrt{(j+2)(2j+1)}}{2j+3}\;(\sqrt{B(GT:\tilde{n},\tilde{\ell},j',\rho\rightarrow\tilde{n},\tilde{\ell},j',\bar{\rho})} + \sqrt{\frac{j+1}{j+2}}g_A), \tag{36}$$

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j,\rho\rightarrow\tilde{n},\tilde{\ell},j,\bar{\rho})} =$$

$$-\frac{(2j+1)}{(2j+3)}\sqrt{\frac{j+2}{j}}\;(\sqrt{B(GT:\tilde{n},\tilde{\ell},j',\rho\rightarrow\tilde{n},\tilde{\ell},j',\bar{\rho})} - \frac{2}{(2j+1)}\sqrt{\frac{j+1}{j+2}}g_A), \tag{37}$$

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j',\rho\rightarrow\tilde{n},\tilde{\ell},j,\bar{\rho})} = \sqrt{B(GT:\tilde{n},\tilde{\ell},j',\bar{\rho}\rightarrow\tilde{n},\tilde{\ell},j,\rho)}. \tag{38}$$

This last relation, (38), also follows from isospin symmetry as well, but if pseudospin symmetry is conserved than the relation holds even though isospin may be violated; i.e., $f_{\tilde{n},\tilde{\ell},\pi} \neq f_{\tilde{n},\tilde{\ell},\nu}$.

IV. AN EXAMPLE: ^{39}K, ^{39}CA

The nuclei $^{39}_{19}\mathrm{K}_{20}$ and $^{39}_{20}\mathrm{Ca}_{19}$ are mirror nuclei. The ground state and first excited state of $^{39}_{19}\mathrm{K}_{20}$ are interpreted as a $0d_{3/2}$ and $1s_{1/2}$ proton hole respectively, while the ground state and first excited state of $^{39}_{20}\mathrm{Ca}_{19}$ are interpreted as a $0d_{3/2}$ and $1s_{1/2}$ neutron hole respectively. These states are members of the $\tilde{n} = 1, \tilde{\ell} = 1$ pseudospin doublet. The M1

transitions between these two states in both of these nuclei have been measured, although they are forbidden in a non-relativistic single-nucleon model, and are indeed small [26,20]. The magnetic moments of the ground states are known. However, the magnetic moments of the excited states are not known so the magnetic relationships introduced in (25) can not be tested at this time.

On the other hand, the Gamow - Teller transitions from the ground state of ^{39}Ca to the ground and first excited state of ^{39}K are known as indicated in Figure 2, which is enough information to test (36). For this example, $j = 1/2$, (36) beomes

$$\sqrt{B(GT:\tilde{1},\tilde{1},3/2^+,\nu\rightarrow\tilde{1},\tilde{1},1/2^+,\pi)} = \frac{\sqrt{5}}{4}\ (\sqrt{B(GT:\tilde{1},\tilde{1},3/2^+,\nu\rightarrow\tilde{1},\tilde{1},3/2^+,\pi)} + \sqrt{0.6}g_A). \tag{39}$$

Of course only the $B(GT)$ is measured; the sign of the square root is unkown. However, we choose the negative sign, $\sqrt{B(GT:\tilde{1},\tilde{1},3/2^+,\nu\rightarrow\tilde{1},\tilde{1},3/2^+,\pi)}_{exp} = -0.647(10)$ [26], because in the non-relativistic limit given in (33), the square root is negative, which also agrees with shell model calculations [20]. Since we are dealing with a single - nucleon model we can expect renormalization of the coupling constant g_A due to omitted shell model configurations just as in the non-relativistic shell model [27]. In Table 1 we see that the quenching necessary to reproduce the experimental "ℓ forbidden" transition $\sqrt{B(GT:\tilde{1},\tilde{1},3/2^+,\nu\rightarrow\tilde{1},\tilde{1},1/2^+,\pi)}_{exp}$ is consistent with the quenching needed in the non-relativistic shell model to reproduce ℓ allowed Gamow - Teller transitions. In the non-relativistic shell model an effective tensor term $g_{eff}\ [Y_2\sigma]^{(1)}$ is added to the Gamow-Teller operator, where Y_2 is the spherical harmonic of rank two and $[\ldots]^{(1)}$ means coupled to angular momentum rank unity. Using a calculated effective coupling constant g_{eff} which includes core polarization, isobar excitations, meson exchange currents, and relativistic corrections, a value of the "ℓ forbidden" transition $\sqrt{B(GT:\tilde{1},\tilde{1},3/2^+,\nu\rightarrow\tilde{1},\tilde{1},1/2^+,\pi)}_{NR} = -0.036(18)$ is calculated. This value agrees with the experimental value within the limits of experi-

mental and theoretical uncertainty. However, the isoscalar and isovector magnetic dipole transitions calculated between the same states and using the same model disagrees with the experimental transitions by a factor of four to five [26]. A measurement of the magnetic moments of the $s_{1/2}$ excited states in ^{39}K and ^{39}Ca would allow the prediction of the forbidden magnetic dipole transitions via (25) which may be helpful in throwing light on this dilemma.

We can now predict the $1/2^+ \rightarrow 1/2^+$ transition using (37). The results are tabulated in Table 2; this transition is the largest within the doublet. Furthermore, the final transition, which is also "ℓ forbidden", can be determined from (29) and (38):

$$\sqrt{B(GT:\tilde{n},\tilde{\ell},j=1/2^+,\nu\rightarrow\tilde{n},\tilde{\ell},j'=3/2^+,\pi)} =$$

$$-\sqrt{2}\sqrt{B(GT:\tilde{n},\tilde{\ell},j=3/2^+,\nu\rightarrow\tilde{n},\tilde{\ell},j'=1/2^+,\pi)} = \mp 0.034(1). \tag{40}$$

This relationship does not depend on the effective g_A but also follows from isospin symmetry as well.

V. CONCLUSIONS

Recent investigations suggest that pseudospin symmetry appears to be only slightly broken particularly near the Fermi sea [2–4,10,9,5]. The empirical evidence for pseudospin symmetry has been in the small energy splittings between doublets. In this paper we analyzed magnetic dipole properties and Gamow-Teller transitions under assumption that pseudospin symmetry is conserved. Pseudospin conservation implies that the spatial wavefunctions of the lower component of the Dirac single - nucleon wavefunction are equal and opposite in sign for pseudospin doublets. Using this assumption, we derive, for spherical nuclei, a relationship for the scalar (vector) magnetic dipole transition between the two states of the doublet and the scalar (vector) magnetic moments of the two states in the doublet. Under

the same assumptions we derive relationships between any two Gamow-Teller transitions from states in the doublet to states in the doublet. We applied the Gamow-Teller relation to the "ℓ forbidden" β - decay of ^{39}Ca, and conclude that agreement occurs for a quenching of the axial coupling constant comparable to that neccessary to fit ℓ allowed Gamow-Teller transitions in the non-relativistic shell model [27,28]. We point out that a measurement of the magnetic moments of the $s_{1/2}$ excited states in ^{39}K and ^{39}Ca would allow the prediction of the forbidden magnetic dipole transitions via (25) which may be helpful in throwing light on an inconsistency posed by the non-relativistic shell model [20]. Furthermore we predict the other two Gamow-Teller transitions from the $1s_{1/2}, 1d_{3/2}$ states in ^{39}Ca to their isobaric analogues in ^{39}K using pseudospin symmetry, thereby producing a test of the effect of pseudospin symmetry on the relativistic single - nucleon wavefunctions.

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TABLES

TABLE I. Predicted "ℓ forbidden" Gamow - Teller strength, $^{39}Ca \rightarrow {}^{39}K$, for various values of the effective axial coupling constant.

$\tilde{g}_A$	$\sqrt{B(GT:\tilde{1},\tilde{1},3/2^+,\nu \rightarrow \tilde{1},\tilde{1},1/2^+,\pi)}$
1.2670 (35) (FREE)	0.187 (6)
0.96 (4) Ref [27]	0.053 (17)
0.91 (2) Ref [28]	0.032 (10)
0.891 (FIT)	0.024 (6)
EXP Ref [20]	$\pm$ 0.024 (1)

TABLE II. Predicted Gamow - Teller strength, $^{39}Ca \rightarrow {}^{39}K$, for two values of the effective effective axial coupling constant.

$\tilde{g}_A$	$\sqrt{B(GT:\tilde{1},\tilde{1},1/2^+,\nu \rightarrow \tilde{1},\tilde{1},1/2^+,\pi)}$
1.2670 (35) (FREE)	1.820 (7)
0.891	1.495 (7)

FIGURES

FIG. 1. Examples of pseudospin doublets in the ^{208}Pb region. n_r is the radial quantum number of the state with $j = \ell + 1/2 = \tilde{\ell} - 1/2$, and is equivalent to $\tilde{n}$, $\tilde{n} = n_r$, ℓ is the orbital angular momentum, j the total angular momentum.

FIG. 2. Measured Gamow - Teller transitions between pseudospin doublets for ^{39}Ca. Dashed line is for "ℓ forbidden" transition

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Examples of Spherical Pseudospin Doublets

$$(n_r \ \ell \, j; (n_r - 1) \, \ell + 2, \, j \ + 1 \ )$$

$$\mathrm{j} = \tilde{\ell} \ \pm \tilde{s} \ \ , \ \tilde{s} \ = 1/2 \ ;$$

$\tilde{\ell}$ pseudo-orbital angular momentum, $\tilde{s}$ pseudo-spin

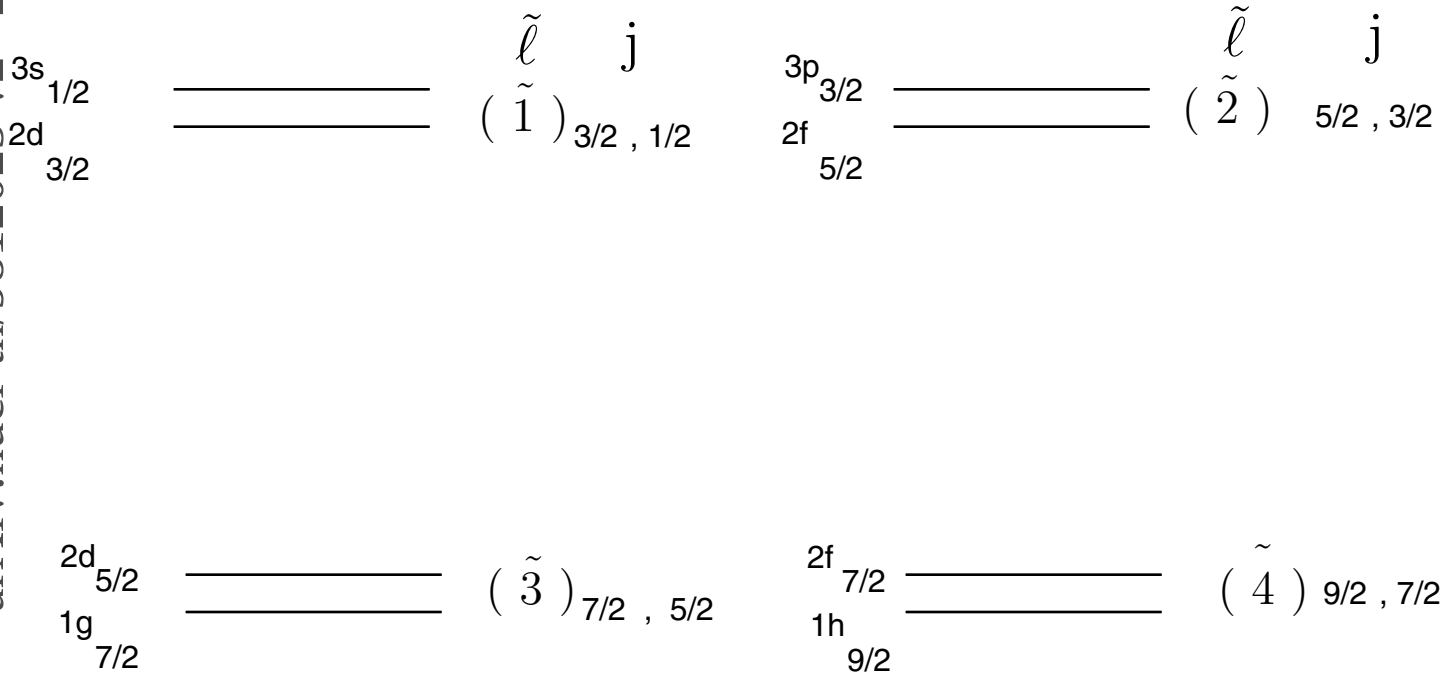

Figure 1

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Gamow - Teller Transitions

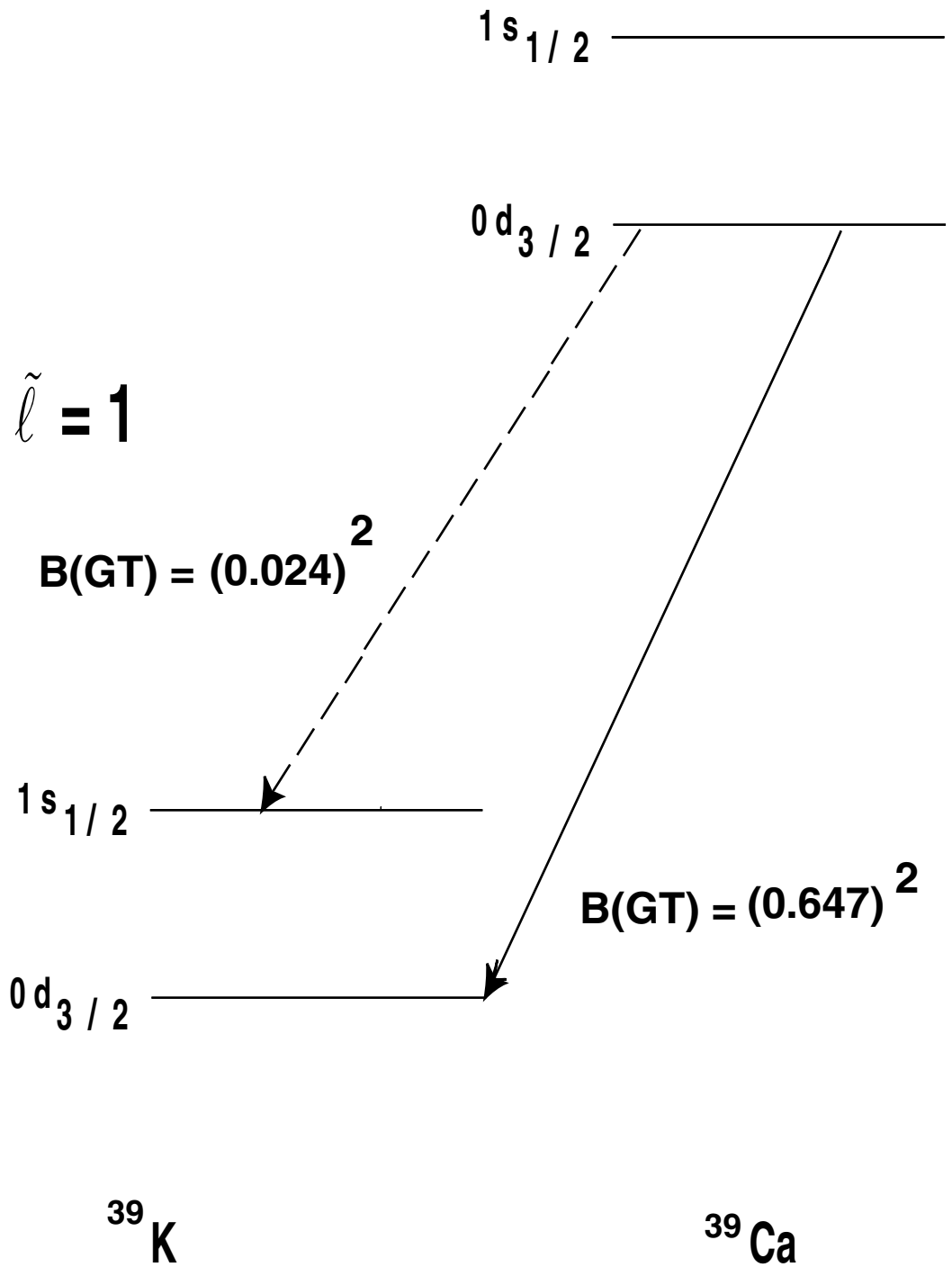

Figure 2