



Heavy Quarks

Nuclear Physics Summer School

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Reference

- A.V. Manohar and M.B. Wise,
Heavy Quark Physics,
Cambridge University Press (2000)

Introduction

QCD describes the dynamics of quarks, and has a non-perturbative scale $\Lambda_{\text{QCD}} \sim 200 \text{ MeV}$.

Simplifications when $m_Q \gg \Lambda_{\text{QCD}}$

A **single** heavy quark interacting with light particles can be described by an effective field theory known as **HQET**.

Applied to c and b quarks. The t -quark decays via $t \rightarrow bW$ before it forms hadrons. The width in the standard model is $\Gamma_t \approx 1.5 \text{ GeV}$

NRQCD

Systems with two heavy quarks (such as J/ψ , Υ or $t\bar{t}$ near threshold) are described by a **completely different** effective theory, NRQCD [non-relativistic QCD].

Velocity Superselection Rule

Consider a heavy quark Q interacting with light degrees of freedom, such as light quarks and gluons.

$$v^\mu = \frac{p^\mu}{m_Q}, \quad \delta v^\mu = \frac{\delta p^\mu}{m_Q} \sim \frac{\Lambda_{\text{QCD}}}{m_Q} \rightarrow 0$$

Quark has a constant velocity.

$$[x, v] = \frac{1}{m_Q} [x, p] = \frac{i\hbar}{m_Q} \rightarrow 0.$$

Cannot simultaneously have a well-defined position and momentum, but can have a well-defined position and velocity.

Spin-Flavor Symmetry

In the $m_Q \rightarrow \infty$ limit: static color 3 source in the rest frame $v^\mu = (1, 0, 0, 0)$.

Strong interactions flavor blind

\Rightarrow HQ flavor symmetry

Symmetry breaking $\propto 1/m_b - 1/m_c$

Color coupling is color electric charge. The magnetic interaction is $\propto 1/m_Q$ for a pointlike spin-1/2 fermion (not true for the proton). \Rightarrow HQ spin symmetry

Symmetry breaking $\propto 1/m_Q$.

Combining gives HQ **$SU(4)$ spin-flavor symmetry**:

$b \uparrow, b \downarrow, c \uparrow$ and $c \downarrow$ transform as a 4

QCD Lagrangian

$$\mathcal{L} = \sum_{i=c,b,t} \bar{Q}_i (i \not{D} - m_{Q_i}) Q_i + \mathcal{L}_{\text{light}}$$

The Lagrangian has m_Q term, and no well-defined $m_Q \rightarrow \infty$ limit. It describes the interactions of Q at all energies, including those greater than m_Q .

Want to consider an effective theory valid for momenta smaller than m_Q , which makes the implications of low momentum manifest.

Should have an expansion in $1/m_Q$

HQET Lagrangian

HQET Lagrangian:

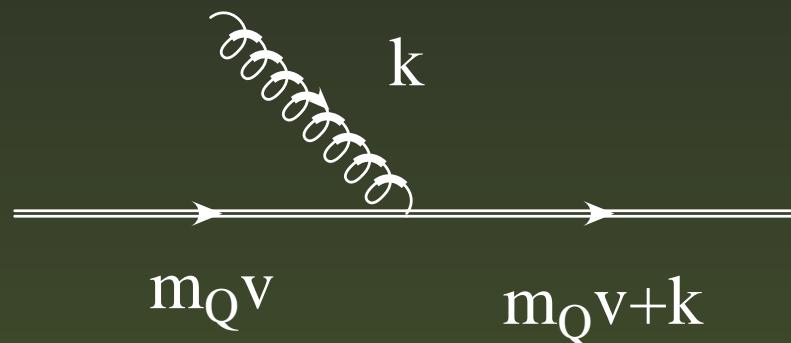
$$\mathcal{L}_{\text{HQET}} = \mathcal{L}_0 + \frac{1}{m_Q} \mathcal{L}_1 + \frac{1}{m_Q^2} \mathcal{L}_2 + \dots$$

\mathcal{L}_0 has spin-flavor symmetry,

$1/m_Q$ terms are symmetry breaking corrections.

Quark Propagator

Look at the quark propagator:



$$i \frac{\not{p} + m_Q}{p^2 - m_Q^2 + i\epsilon}$$

$p = mv + k$, where k is called the residual momentum, and is of order Λ_{QCD} .

HQET Propagator

$$i \frac{m_Q \not{\psi} + \not{k} + m_Q}{(m_Q v + \not{k})^2 - m_Q^2 + i\epsilon}$$

Expanding this in the limit $\not{k} \ll m_Q$ gives

$$i \frac{1 + \not{\psi}}{2k \cdot v + i\epsilon} + \mathcal{O}\left(\frac{k}{m_Q}\right) = i \frac{P_+}{k \cdot v + i\epsilon} + \mathcal{O}\left(\frac{k}{m_Q}\right),$$

with a well defined limit.

$$P_+ \equiv \frac{1 + \not{\psi}}{2}$$

Projectors

$$P_+ = \frac{1 + \psi}{2}, \quad P_- = \frac{1 - \psi}{2},$$

In the rest frame,

$$P_+ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_+^2 = P_+, \quad P_-^2 = P_-, \quad P_+ + P_- = 1, \quad P_+ P_- = 0, \quad P_- P_+ = 0.$$

Gluon Vertex

The quark-gluon vertex

$$-igT^a\gamma^\mu \rightarrow -igT^a P_+ \gamma^\mu P_+ = -igT^a v^\mu,$$

using the identity

$$P_+ \gamma^\mu = v^\mu + \gamma^\mu P_-$$

In the rest frame: the coupling is purely that of an electric charge.

HQET L_0

HQET Lagrangian:

$$\mathcal{L} = \bar{h}_v(x) (iD \cdot v) h_v(x),$$

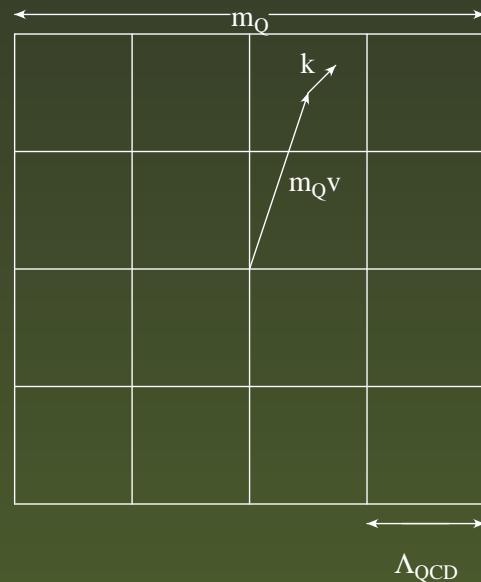
$h_v(x)$ is the quark field in the effective theory and satisfies

$$P_+ h_v(x) = h_v(x).$$

h_v annihilates quarks with velocity v , but **does not** create antiquarks

Dividing up momentum space

v appears explicitly in the HQET Lagrangian.
 h_v describes quarks with velocity v , and momenta
within Λ_{QCD} of $m_Q v$.



quarks with velocity $v' \neq v$ are far away in the EFT

Feynman rules

$$\mathcal{L} = \bar{h}_v (iD \cdot v) h_v$$

$$D_\mu = \partial_\mu + igT^a A_\mu^a$$

The $\partial \cdot v$ term gives a propagator

$$\frac{iP_+}{k \cdot v}$$

The $A \cdot v$ term gives a vertex

$$-igT^a v^\mu$$

Manifest Spin-Flavor Symmetry

$$\mathcal{L}_0 = \sum_{f=c,b,t} \bar{h}_{fv} (iD \cdot v) h_{fv},$$

has manifest spin-flavor symmetry, since $D \cdot v$ does not depend on the spin or the flavor of the heavy quark.

Light degrees of freedom

Hadrons containing a single heavy quark contain Q , and light quarks and gluons [light degrees of freedom ℓ].

D^+ meson has a c quark, \bar{d} quark, plus $\bar{q}q$ pairs and gluons. Quantum numbers of ℓ are the same as the \bar{d} .

Total angular momentum \mathbf{J} is conserved

\mathbf{S}_Q is conserved as $m_Q \rightarrow \infty$

Define $\mathbf{S}_\ell \equiv \mathbf{J} - \mathbf{S}_Q$

Spin of the light degrees of freedom

Multiplet Structure

$$\mathbf{J}^2 = j(j+1), \quad \mathbf{S}_Q^2 = s_Q(s_Q+1), \quad \mathbf{S}_\ell^2 = s_\ell(s_\ell+1)$$

$s_Q = 1/2$, so heavy hadrons are in degenerate multiplets with $j = s_\ell \pm 1/2$, unless $s_\ell = 0$, in which case there is a single $j = 1/2$ multiplet.

Ground state mesons: Q and a light antiquark \bar{q} , so $s_\ell = 1/2$.

$j = 0 \oplus 1$ and negative parity, since quarks and antiquarks have opposite parity

Degenerate 0^- and 1^- mesons which form a flavor $\bar{\mathbf{3}}$

Called $H^{(Q)}$

Ground State Mesons

D_s^+, D_s^{*+}



D^0, D^{*0}



D^+, D^{*+}



$\bar{B}_s^0, \bar{B}^-, \bar{B}^0$ (spin-0) $\bar{B}_s^{*0}, \bar{B}^{*-}, \bar{B}^{*0}$ (spin-1)

NOTE: $c \in D$, $b \in \bar{B}$

Ground State Baryons

One Q and qq . Either (anti)symmetric in both flavor and spin.

$$\text{Spin } \left(\frac{1}{2} \times \frac{1}{2}\right)_S = 1 \quad \text{Flavor } (3 \times 3)_S = 6$$

$$\text{Spin } \left(\frac{1}{2} \times \frac{1}{2}\right)_A = 0 \quad \text{Flavor } (3 \times 3)_A = \bar{3}$$

$s_\ell = 0 \Rightarrow j = 1/2$ baryons which are flavor $\bar{3}$

$s_\ell = 1 \Rightarrow j = 1/2, 3/2$ baryons which are flavor 6

$$\Lambda^{(Q)} \quad \Sigma^{(Q)}, \Sigma^{*(Q)}$$

3 Baryons

 Λ_c^+ 

cud

 Ξ_c^0 

cds

 Ξ_c^+ 

cus

 $\Lambda_b^0, \Xi_b^-, \Xi_b^0$

6 Baryons

$$\Sigma_c^0, \Sigma_c^{*0}$$



cdd

$$\Sigma_c^+, \Sigma_c^{*+}$$



cud

$$\Sigma_c^{++}, \Sigma_c^{*++}$$



cuu

$$\Xi_c'{}^0, \Xi_c^{*0}$$



cds

$$\Xi_c'{}^+, \Xi_c^{*+}$$



cus

$$\Omega_c^0, \Omega_c^{*0}$$



css

$$\Sigma_b^{-,0,+}, \Xi_b^{-,0}, \Omega_b^-$$

Quark Field

Write the quark field as

$$\begin{aligned} Q(x) &= e^{-im_Q v \cdot x} [h_v(x) + Q_v(x)] \\ &= e^{-im_Q t} \begin{pmatrix} h_v(x) \\ Q_v(x) \end{pmatrix} \end{aligned}$$

$p = m_Q v + k$, so the x dependence of $h_v(x)$ is k .

$$\begin{aligned} \not{p} h_v(x) &= h_v(x), \\ \not{p} Q_v(x) &= -Q_v(x) \end{aligned}$$

Tree-level Lagrangian

At tree-level, one does not have to worry about renormalization effects:

$$\begin{aligned}
 \mathcal{L} &= \bar{Q} (i \not{D} - m_Q) Q \\
 &= (\bar{h}_v + \bar{\mathcal{Q}}_v) e^{im_Q v \cdot x} (i \not{D} - m_Q) e^{-im_Q v \cdot x} (h_v + \mathcal{Q}_v) \\
 &= (\bar{h}_v + \bar{\mathcal{Q}}_v) (i \not{D} - m_Q + m_Q \not{\psi}) (h_v + \mathcal{Q}_v) \\
 &= (\bar{h}_v + \bar{\mathcal{Q}}_v) (i \not{D} h_v + [i \not{D} - 2m_Q] \mathcal{Q}_v)
 \end{aligned}$$

The result can be simplified using

$$P_+ \gamma^\mu P_+ = v^\mu, \quad P_- \gamma^\mu P_- = -v^\mu, \quad P_+ \gamma^\mu P_- = \gamma_\perp^\mu, \quad P_- \gamma^\mu P_+ = \gamma_\perp^\mu,$$

where the \perp projector is defined for any vector A by

$$A_\perp^\mu \equiv A^\mu - v^\mu v \cdot A.$$

$$\mathcal{L} = \bar{h}_v (iv \cdot D) h_v - \mathcal{Q}_v (iv \cdot D + 2m_Q) \mathcal{Q}_v + \bar{h}_v i \not{\partial}_{\perp} \mathcal{Q}_v + \bar{\mathcal{Q}}_v i \not{\partial}_{\perp} h_v$$

Quadratic in \mathcal{Q}_v :

$$\mathcal{L} = \bar{h}_v (iv \cdot D) h_v + \bar{h}_v i \not{\partial}_{\perp} \frac{1}{2m_Q + iv \cdot D} i \not{\partial}_{\perp} h_v.$$

The last term can be expanded in a power series in $1/m_Q$,

$$\frac{1}{2m_Q + iv \cdot D} = \frac{1}{2m_Q} - \frac{1}{4m_Q^2} iv \cdot D + \dots .$$

$1/m_Q$ Lagrangian

The effective Lagrangian to order $1/m_Q$ is thus

$$\mathcal{L} = \bar{h}_v (iv \cdot D) h_v + \frac{1}{2m_Q} \bar{h}_v i \not{D}_\perp i \not{D}_\perp h_v.$$

This can be rewritten using the identities

$$\gamma^\alpha \gamma^\beta = g^{\alpha\beta} - i\sigma^{\alpha\beta}, \quad [D^\alpha, D^\beta] = igG^{\alpha\beta}$$

$$\begin{aligned} \mathcal{L} &= \bar{h}_v (iv \cdot D) h_v + \frac{1}{2m_Q} \bar{h}_v (iD_\perp)^2 h_v - \frac{g}{4m_Q} \bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v \\ &\quad + \mathcal{O}\left(\frac{1}{m_Q^2}\right) \end{aligned}$$

$$\mathcal{L} = \bar{h}_v (iv \cdot D) h_v + \frac{1}{2m_Q} \bar{h}_v (iD_{\perp})^2 h_v - c_F \frac{g}{4m_Q} \bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v$$

The $(iD_{\perp})^2$ term violates flavor symmetry at order $1/m_Q$

$g\sigma_{\alpha\beta}G^{\alpha\beta}$ term violates spin and flavor symmetry at order $1/m_Q$

One can carry out the expansion to higher order in $1/m_Q$ to obtain the tree level HQET Lagrangian.

Masses

Hadron mass mass in effective theory is $M_H - m_Q$.

- Lowest order: all hadrons degenerate, mass m_Q
- Order one: Hadron mass

$$\langle H_Q | \mathcal{H}_0 | H_Q \rangle \equiv \bar{\Lambda}$$

where H_0 = Hamiltonian from lowest order Lagrangian (including the light degrees of freedom).

$\bar{\Lambda}$ has different values for each multiplet:
 $\bar{\Lambda}$ for mesons, $\bar{\Lambda}_\Lambda$, $\bar{\Lambda}_\Sigma$

λ_1, λ_2

■ Order $1/m_Q$:

$$\frac{\mathcal{H}_1}{m_Q} = -\frac{\mathcal{L}_1}{m_Q} = -\frac{1}{2m_Q} \bar{h}_v (iD_{\perp})^2 h_v + c_F \frac{g}{4m_Q} \bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v.$$

Define two non-perturbative parameters

$$\begin{aligned}\lambda_1 &= \langle H_Q | \bar{h}_v (iD_{\perp})^2 h_v | H_Q \rangle, \\ 8(\mathbf{S}_Q \cdot \mathbf{S}_{\ell}) \lambda_2 &= \langle H_Q | \bar{h}_v g \sigma_{\alpha\beta} G^{\alpha\beta} h_v | H_Q \rangle,\end{aligned}$$

In the rest frame:

$$\begin{aligned}(iD_{\perp})^2 &= -\mathbf{p}^2 \\ \sigma_{\alpha\beta} G^{\alpha\beta} &= -2\boldsymbol{\sigma} \cdot \mathbf{B}\end{aligned}$$

Meson Masses

$$m_H = m_Q + \bar{\Lambda} - \frac{\lambda_1}{2m_Q} + c_F \frac{2\lambda_2 \mathbf{S}_Q \cdot \mathbf{S}_\ell}{m_Q}$$

$$\mathbf{S}_Q \cdot \mathbf{S}_\ell = (J^2 - S_Q^2 - S_\ell^2)/2, \text{ so}$$

$$m_B = m_b + \bar{\Lambda} - \frac{\lambda_1}{2m_b} - \frac{3\lambda_2}{2m_b}$$

$$m_{B^*} = m_b + \bar{\Lambda} - \frac{\lambda_1}{2m_b} + \frac{\lambda_2}{2m_b}$$

$$m_D = m_c + \bar{\Lambda} - \frac{\lambda_1}{2m_c} - \frac{3\lambda_2}{2m_c}$$

$$m_{D^*} = m_c + \bar{\Lambda} - \frac{\lambda_1}{2m_c} + \frac{\lambda_2}{2m_c}$$

Note that heavy quark symmetry implies that $\bar{\Lambda}$, λ_1 and λ_2 have the same value in the b and c systems (upto renormalization)

$$0.49 \text{ GeV}^2 = m_{B^*}^2 - m_B^2 = 4\lambda_2 = m_{D^*}^2 - m_D^2 = 0.55 \text{ GeV}^2,$$

up to corrections of order $1/m_{b,c}$.

$$90 \pm 3 \text{ MeV} = m_{B_s} - m_{B_d} = \bar{\Lambda}_s - \bar{\Lambda}_d = m_{D_s} - m_{D_d} = 99 \pm 1 \text{ MeV}$$

$$345 \pm 9 \text{ MeV} = m_{\Lambda_b} - m_B = \bar{\Lambda}_\Lambda - \bar{\Lambda}_d = m_{\Lambda_c} - m_D = 416 \pm 1 \text{ MeV}$$

$\bar{\Lambda}$, λ_1 and λ_2 will occur elsewhere

c-Meson Spectrum

	(MeV)		J^P	s_ℓ
D^+	1869.3 ± 0.5	$c\bar{d}$	0^-	$1/2$
D^{*+}	2010.0 ± 0.5		1^-	
D^0	1864.6 ± 0.5	$c\bar{u}$	0^-	$1/2$
D^{*0}	2006.7 ± 0.5		1^-	
D_s^+	1968.5 ± 0.6	$c\bar{s}$	0^-	$1/2$
D_s^{*+}	2112.4 ± 0.7		1^-	
D_0^*			0^+	$1/2$
D_1^*	2461 ± 50	$c\bar{q}$	1^+	
D_1	2422.2 ± 1.8	$c\bar{q}$	1^+	$3/2$
D_2^*	2458.9 ± 2.0		2^+	

c-Baryon Spectrum

Λ_c^+	2284.9 ± 0.6	$c[ud]$	$1/2^+$	0
Ξ_c^+	2465.6 ± 1.4	$c[us]$	$1/2^+$	0
Ξ_c^0	2470.3 ± 1.8	$c[ds]$	$1/2^+$	0
Σ_c^{++}	2452.8 ± 0.6	$c(uu)$	$1/2^+$	1
Σ_c^{*++}	2519.4 ± 1.5		$3/2^+$	
Σ_c^+	2453.6 ± 0.9	$c(ud)$	$1/2^+$	1
Σ_c^{*+}			$3/2^+$	
Σ_c^0	2452.2 ± 0.6	$c(dd)$	$1/2^+$	1
Σ_c^{*0}	2517.5 ± 1.4		$3/2^+$	
$\Xi_c^{\prime +}$	2573.4 ± 3.3	$c(us)$	$1/2^+$	1
Ξ_c^{*+}	2644.6 ± 2.1		$3/2^+$	
$\Xi_c^{\prime 0}$	2577.3 ± 3.4	$c(ds)$	$1/2^+$	1
Ξ_c^{*0}	2643.8 ± 1.8		$3/2^+$	
Ω_c^0	2704 ± 4	$c(ss)$	$1/2^+$	1
Ω_c^{*0}			$3/2^+$	

b-Meson Spectrum

	(MeV)		J^P	s_ℓ
\bar{B}^0	5279.2 ± 1.8	$b\bar{d}$	0^-	$1/2$
\bar{B}^{*0}	5324.9 ± 1.8		1^-	
\bar{B}^-	5278.9 ± 1.8	$b\bar{u}$	0^-	$1/2$
\bar{B}^{*-}	5324.9 ± 1.8		1^-	
\bar{B}_s^0	5369.3 ± 2.0	$b\bar{s}$	0^-	$1/2$
\bar{B}_s^{*0}			1^-	
\bar{B}_0^*		$b\bar{q}$	0^+	$1/2$
\bar{B}_1^*			1^+	
\bar{B}_1		$b\bar{q}$	1^+	$3/2$
\bar{B}_2^*			2^+	

b-Baryon Spectrum

Λ_b^0	5624 ± 9	$b[ud]$	$1/2^+$	0
Ξ_b^0		$b[us]$	$1/2^+$	0
Ξ_b^-		$b[ds]$	$1/2^+$	0
Σ_b^+			$1/2^+$	1
Σ_b^{*+}		$b(uu)$	$3/2^+$	
Σ_b^0			$1/2^+$	1
Σ_b^{*0}		$b(ud)$	$3/2^+$	
Σ_b^-			$1/2^+$	1
Σ_b^{*-}		$b(dd)$	$3/2^+$	
$\Xi_b'^0$			$1/2^+$	1
Ξ_b^{*0}		$b(us)$	$3/2^+$	
$\Xi_b'^-$			$1/2^+$	1
Ξ_b^{*-}		$b(ds)$	$3/2^+$	
Ω_b^-			$1/2^+$	1
Ω_b^{*-}		$b(ss)$	$3/2^+$	

Meson Field

$Q\bar{q}$ mesons \rightarrow field $H_v^{(Q)}$ (4×4 matrix, bispinor)

Pseudoscalar $P_v^{(Q)}(x)$ and vector $P_{v\mu}^{*(Q)}(x)$

Vector particles have a polarization vector ϵ_μ , with $\epsilon \cdot \epsilon = -1$, and $v \cdot \epsilon = 0$.

$$H_v^{(Q)} = \frac{1 + \not{v}}{2} \left[\not{P}_v^{*(Q)} + i P_v^{(Q)} \gamma_5 \right].$$

$$H_v^{(Q)} = \frac{1 + \not{v}}{2} \left[\not{P}_v^{*(Q)} + i P_v^{(Q)} \gamma_5 \right].$$

$$\not{v} H_v^{(Q)} = H_v^{(Q)}, \quad H_v^{(Q)} \not{v} = -H_v^{(Q)}.$$

using $v \cdot P_v^{*(Q)} = 0$

Conjugate field:

$$\bar{H}_v^{(Q)} = \gamma^0 H_v^{(Q)\dagger} \gamma^0 = \left[\not{P}_v^{*(Q)\dagger} + i P_v^{(Q)\dagger} \gamma_5 \right] \frac{1 + \not{\psi}}{2},$$

which also transforms as a bispinor,

$$H_{v_r}^{(Q)} = \begin{pmatrix} 0 & iP_{v_r}^{(Q)} - \boldsymbol{\sigma} \cdot \mathbf{P}_{v_r}^{*(Q)} \\ 0 & 0 \end{pmatrix},$$

$$\begin{aligned} \mathbf{S}_Q H_{v_r}^{(Q)} &= \frac{1}{2} \boldsymbol{\sigma}_{4 \times 4} H_{v_r}^{(Q)} \\ \mathbf{S}_\ell H_{v_r}^{(Q)} &= -\frac{1}{2} H_{v_r}^{(Q)} \boldsymbol{\sigma}_{4 \times 4} \end{aligned}$$

Rotations:

$$\delta P_{v_r}^{(Q)} = 0, \quad \delta \mathbf{P}_{v_r}^{(Q)} = \delta \boldsymbol{\theta} \times \mathbf{P}_{v_r}^{*(Q)}$$

HQ Spin:

$$\delta P_{v_r}^{(Q)} = -\frac{1}{2}\delta\theta \cdot \mathbf{P}_{v_r}^{(Q)}, \quad \delta\mathbf{P}_{v_r}^{*(Q)} = \frac{1}{2}\delta\theta \times \mathbf{P}_{v_r}^{*(Q)} - \frac{1}{2}\delta\theta P_{v_r(Q)}.$$

Under finite heavy quark spin transformations,

$$H_v^{(Q)} \rightarrow D(R)_Q H_v^{(Q)},$$

where $D(R)$ is the rotation matrix in the spinor representation for the rotation R .

Normalization of States

$$\langle H(p', \varepsilon') | H(p, \varepsilon) \rangle = 2E_{\mathbf{p}} (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta_{\varepsilon \varepsilon'},$$

Mass dimension -1

HQET states eigenstates of the $m_Q \rightarrow \infty$ theory and labelled by v and k , with $v \cdot k = 0$. They differ from full QCD states.

$$\langle H(v', k', \varepsilon') | H(v, k, \varepsilon) \rangle = 2v^0 (2\pi)^3 \delta_{vv'} \delta^3(\mathbf{k} - \mathbf{k}') \delta_{\varepsilon \varepsilon'}.$$

Usually take $k = 0$. States have mass dimension $-3/2$

$$|H(p)\rangle = \sqrt{m_H} \left[|H(v)\rangle + \mathcal{O}\left(\frac{1}{m_Q}\right) \right]$$

Similarly

$$\begin{aligned}\bar{u}(p, s)\gamma^\mu u(p, s) &= 2p^\mu \\ \bar{u}(v, s)\gamma^\mu u(v, s) &= 2v^\mu \\ u(p, s) &= \sqrt{m_H}u(v, s)\end{aligned}$$

$\bar{B} \rightarrow D^{(*)}$ Form Factors

Semileptonic $b \rightarrow c$ decays via the weak current $\bar{c} \gamma_\mu P_L b$
 Decay form-factors are defined by:

$$\begin{aligned}\langle D(p') | V^\mu | \bar{B}(p) \rangle &= f_+(q^2) (p + p')^\mu + f_-(q^2) (p - p')^\mu, \\ \langle D^*(p', \epsilon) | V^\mu | \bar{B}(p) \rangle &= g(q^2) \epsilon^{\mu\nu\alpha\tau} \epsilon_\nu^* (p + p)_\alpha (p - p)_\tau, \\ \langle D^*(p', \epsilon) | A^\mu | \bar{B}(p) \rangle &= -if(q^2) \epsilon^{*\mu} \\ &\quad -i\epsilon^* \cdot p \left[a_+(q^2) (p + p')^\mu + a_-(q^2) (p - p')^\mu \right],\end{aligned}$$

where $q = p - p'$
 Six form-factors

w

Label states by v and v' , and use

$$w = v \cdot v' = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$$

The allowed kinematic range for w is

$$0 \leq w - 1 \leq \frac{(m_B - m_{D^{(*)}})^2}{2m_B m_{D^{(*)}}}$$

The zero-recoil point, at which $D^{(*)}$ is at rest in the \bar{B} rest frame, is $w = 1$ (maximum q^2)

Better to use:

$$\frac{\langle D(p')|V^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_D}} = h_+(w) (v + v')^\mu + h_-(w) (v - v')^\mu,$$

$$\frac{\langle D^*(p', \epsilon)|V^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_{D^*}}} = h_V(w) \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* v'_\alpha v_\beta,$$

$$\begin{aligned} \frac{\langle D^*(p', \epsilon)|A^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_{D^*}}} &= -i h_{A_1}(w) (w+1) \epsilon^{*\mu} + i h_{A_2}(w) (\epsilon^* \cdot v) v^\mu \\ &\quad + i h_{A_3}(w) (\epsilon^* \cdot v) v'^\mu. \end{aligned}$$

$$q_{\text{light}}^2 \sim (\Lambda_{\text{QCD}} v - \Lambda_{\text{QCD}} v')^2 = 2\Lambda_{\text{QCD}}^2(1-w)$$

HQ symmetry should hold if:

$$2\Lambda_{\text{QCD}}^2 (w-1) \ll m_{b,c}^2.$$

The heavy meson form factors are expected to vary on the scale $q_{\text{light}}^2 \sim \Lambda_{\text{QCD}}^2$, i.e. on the scale $w \sim 1$.

QCD matrix elements are of the form:

$$\langle H^{(c)}(p') | \bar{c} \Gamma b | H^{(b)}(p) \rangle$$

At leading order in $1/m_{c,b}$ and $\alpha_s(m_{c,b})$:

$$\langle H^{(c)}(v') | \bar{c}_{v'} \Gamma b_v | H^{(b)}(v) \rangle$$

use spurion trick $\Gamma \rightarrow D(R)_c \Gamma D(R)_b^{-1}$

$$\bar{c}_{v'} \Gamma b_v = \text{Tr } X \bar{H}_{v'}^{(c)} \Gamma H_v^{(b)},$$

$$X = X_0 + X_1 \not{p} + X_2 \not{p}' + X_3 \not{p} \not{p}',$$

where the coefficients are functions of $w = v \cdot v'$.

Isgur-Wise Function

Use $X = -\xi(w)$:

$$\langle D(v') | \bar{c}_{v'} \gamma_\mu b_v | \bar{B}(v) \rangle = \xi(w) [v_\mu + v'_\mu],$$

$$\langle D^*(v', \epsilon) | \bar{c}_{v'} \gamma_\mu \gamma_5 b_v | \bar{B}(v) \rangle = -i\xi(w) [(1+w)\epsilon_\mu^* - (\epsilon^* \cdot v)v'_\mu],$$

$$\langle D^*(v', \epsilon) | \bar{c}_{v'} \gamma_\mu b_v | \bar{B}(v) \rangle = \xi(w) \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} v'^\alpha v^\beta.$$

Six form-factors in terms of one Isgur-Wise function

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w),$$

$$h_-(w) = h_{A_2}(w) = 0$$

Normalization of $\xi(1)$

Consider the forward matrix element of the vector current $\bar{b}\gamma^\mu b$ between \bar{B} meson states. Setting $v' = v$, and letting $c \rightarrow b$, $D \rightarrow \bar{B}$,

$$\frac{\langle \bar{B}(p) | \bar{b}\gamma_\mu b | \bar{B}(p) \rangle}{m_B} = \langle \bar{B}(v) | \bar{b}_v \gamma_\mu b_v | \bar{B}(v) \rangle = 2 \xi(w=1) v_\mu.$$

where ξ for $b \rightarrow b$ is the same as for $b \rightarrow c$.

So $\xi(1) = 1$.

This fixes the absolute normalization and allows one to determine V_{cb} .

Radiative Corrections

Can be computed in a systematic way.

$\bar{B} \rightarrow D^{(*)}$ at zero-recoil

$$\bar{c}\gamma_\mu b = C_1^{(V)}\bar{c}_{v'}\gamma_\mu b_v + C_2^{(V)}\bar{c}_{v'}v_\mu b_v + C_3^{(V)}\bar{c}_{v'}v'_\mu b_v$$

and

$$\bar{c}\gamma_\mu\gamma_5 b = C_1^{(A)}\bar{c}_{v'}\gamma_\mu\gamma_5 b_v + C_2^{(A)}\bar{c}_{v'}v_\mu\gamma_5 b_v + C_3^{(A)}\bar{c}_{v'}v'_\mu\gamma_5 b_v.$$

At the zero-recoil point:

$$\bar{c}\gamma_\mu b = \eta_V \bar{c}_{v'}\gamma_\mu b_v$$

$$\bar{c}\gamma_\mu\gamma_5 b = \eta_A \bar{c}_{v'}\gamma_\mu\gamma_5 b_v$$

$$\eta_V = 1 + \frac{\alpha_s(\mu)}{\pi} \left[-2 + \left(\frac{m_b + m_c}{m_b - m_c} \right) \ln \left(\frac{m_b}{m_c} \right) \right].$$

$$\eta_A = 1 + \frac{\alpha_s(\mu)}{\pi} \left[-\frac{8}{3} + \frac{(m_b + m_c)}{(m_b - m_c)} \ln \left(\frac{m_b}{m_c} \right) \right].$$

(Known to two-loops)

$1/m$ Corrections

$$\mathcal{L} = \bar{h}_v (iv \cdot D) h_v + \frac{1}{2m_Q} \bar{h}_v (iD_{\perp})^2 h_v - c_F \frac{g}{4m_Q} \bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v$$

$$c_F(\mu) = \left[\frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right]^{9/(33-2n_f)}$$

Reparameterization Invariance

$$\begin{aligned} p_Q &= m_Q v + k, \\ v &\rightarrow v + \varepsilon/m_Q, \\ k &\rightarrow k - \varepsilon. \end{aligned}$$

Since $v^2 = 1$, $v \cdot \varepsilon = 0$.

Also $\psi Q_v = Q_v$ so the change in the field:

$$Q_v \rightarrow Q_v + \delta Q_v,$$

δQ_v satisfies

$$\left(\psi + \frac{\not{v}}{m_Q} \right) (Q_v + \delta Q_v) = Q_v + \delta Q_v.$$

so that

$$(1 - \psi) \delta Q_v = \frac{\not{f}}{m_Q} Q_v.$$

One can choose:

$$\delta Q_v = \frac{\not{f}}{2m_Q} Q_v.$$

[Not unique, one can always make field redefinitions]
 L invariant under

$$\begin{aligned} v &\rightarrow v + \varepsilon/m_Q, \\ Q_v &\rightarrow e^{i\varepsilon \cdot x} \left(1 + \frac{\not{f}}{2m_Q}\right) Q_v, \end{aligned}$$

$$\begin{aligned}\mathcal{L}_0 &\rightarrow \mathcal{L}_0 + \frac{1}{m_Q} \bar{Q}_v (i\varepsilon \cdot D) Q_v, \\ \mathcal{L}_1 &\rightarrow \mathcal{L}_1 - \frac{1}{m_Q} \bar{Q}_v (i\varepsilon \cdot D) Q_v.\end{aligned}$$

so that the kinetic energy is not renormalized.

Other connections that follow from
reparameterization invariance:

$$c_S = 2c_F - 1, \quad \sigma \cdot \nabla \times E$$

E.g. relates matching coefficients of leading order
and $1/m$ operators, and their anomalous dimensions.

Luke's Theorem

Can compute $1/m$ corrections to meson form-factors.
Two sources of $1/m$ corrections, those from the Lagrangian, and from the current. So one has

$$T(\mathcal{L}_1, J_0), \quad J_1$$

where

$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{m} \mathcal{L}_1 + \dots, \quad J = J_0 + \frac{1}{m} J_1 + \dots$$

Luke's theorem: no $1/m$ corrections to the form-factor at zero recoil.

V_{cb}

Experimentally, measure $\bar{B} \rightarrow D^*$ which determines $|V_{cb}\mathcal{F}(1)|$.

$$\mathcal{F}(1) = \eta_A + 0 + \mathcal{O}\left(\frac{1}{m^2}\right)$$

$\eta_A = 0.96$, and $1/m^2 \approx -0.05$, so

$$\mathcal{F}(1) = 0.91 \pm 0.05$$

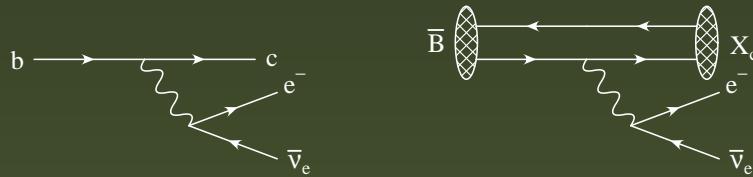
and from this one finds

$$|V_{cb}| = [38.6 \pm 1.5(\text{exp}) \pm 2.0(\text{th})] \times 10^{-3},$$

Inclusive Semileptonic Decay

Inclusive decay due to

$$H_W = \frac{4G_F}{\sqrt{2}} V_{cb} \bar{c} \gamma^\mu P_L b \bar{e} \gamma_\mu P_L \nu_e.$$



The inclusive decay rate is

$$\begin{aligned} \frac{d\Gamma}{dq^2 dE_e dE_{\nu_e}} &= \int \frac{d^4 p_e}{(2\pi)^4} \int \frac{d^4 p_{\nu_e}}{(2\pi)^4} 2\pi\delta(p_e^2) 2\pi\delta(p_{\nu_e}^2) \theta(p_e^0)\theta(p_{\nu_e}^0) \\ &\quad \times \delta(E_e - p_e^0)\delta(E_{\nu_e} - p_{\nu_e}^0)\delta(q^2 - (p_e + p_{\nu_e})^2) \\ &\quad \times \sum_{X_c} \sum_{\text{lepton spins}} \frac{\left| \langle X_c e \bar{\nu}_e | H_W | \bar{B} \rangle \right|^2}{2m_B} (2\pi)^4 \delta^4(p_B - (p_e + p_{\nu_e}) - p_{X_c}), \end{aligned}$$

Calculation similar to deep inelastic scattering. Do an operator product expansion in powers of $1/m_Q$ rather than powers of $1/Q$. [No twist here].

Leading term like free-quark decay

No $1/m_Q$ correction

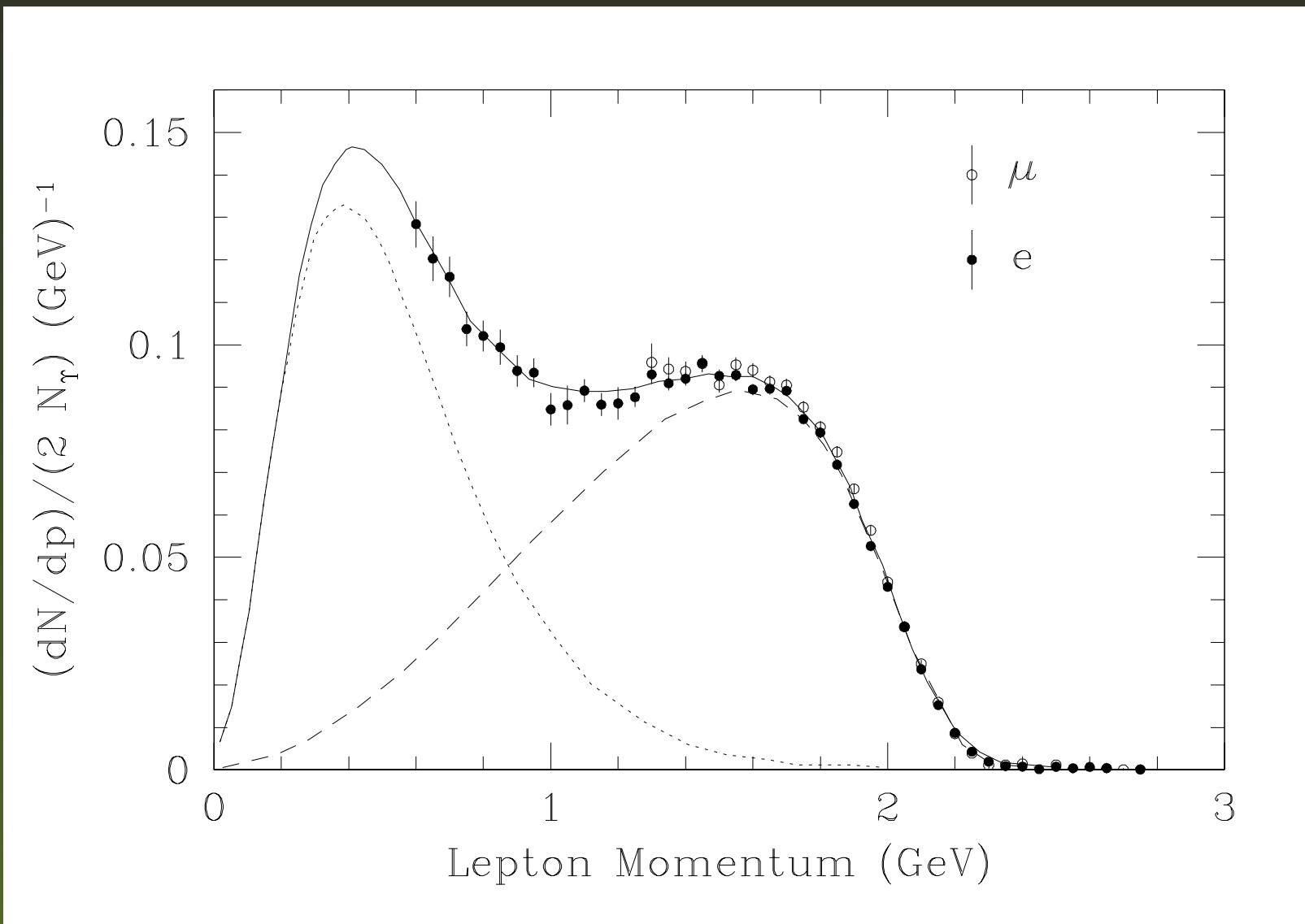
$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[1 + \frac{\lambda_1}{2m_b^2} + \frac{3\lambda_2}{2m_b^2} \left(2\rho \frac{d}{d\rho} - 3 \right) \right] f(\rho),$$

where

$$f(\rho) = 1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln \rho.$$

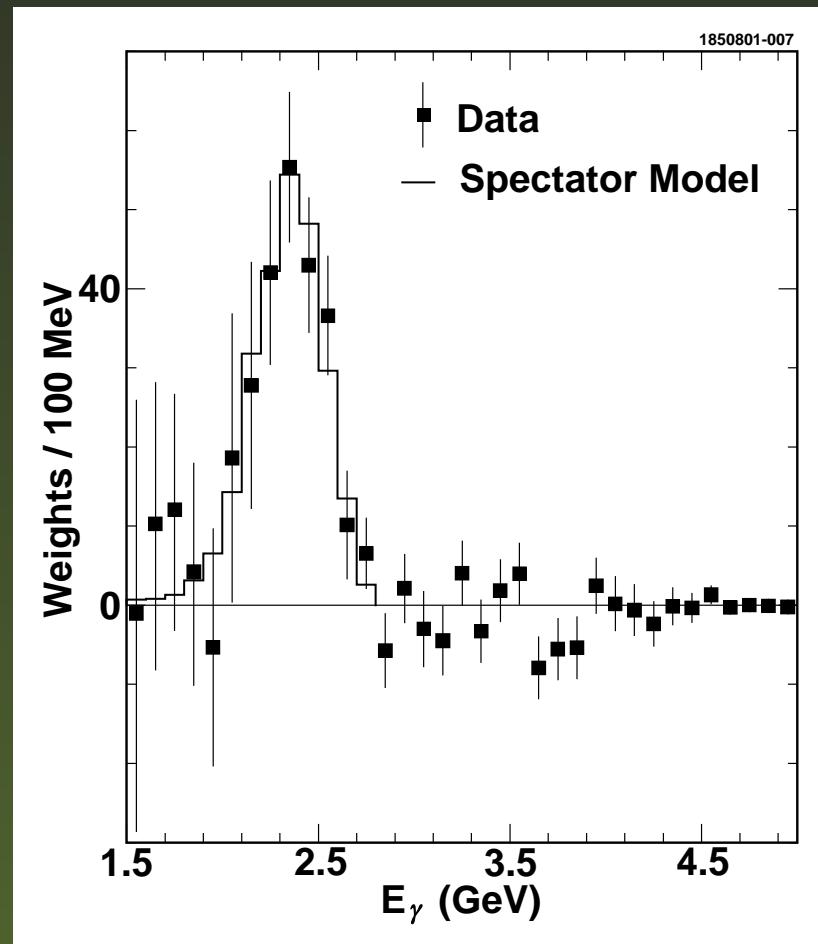
For $b \rightarrow u$:

$$\Gamma = \frac{G_F^2 m_b^5 |V_{ub}|^2}{192\pi^3} \left[1 + \frac{\lambda_1}{2m_b^2} - \frac{9\lambda_2}{2m_b^2} \right].$$



$b \rightarrow s\gamma$

CLEO $b \rightarrow s\gamma$ photon spectrum



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- $\mathcal{B}(b \rightarrow s\gamma) = (3.21 \pm 0.43 \pm 0.27 \pm 0.15) \times 10^{-4}$
 - $\langle E_\gamma \rangle = 2.346 \pm 0.04 \text{ GeV}$
 - $\langle E_\gamma^2 \rangle - \langle E_\gamma \rangle^2 = 0.0226 \pm 0.008 \text{ GeV}$

Standard model BR is $(3.28 \pm 0.33) \times 10^{-4}$

Summary

A lot of high precision data to come.